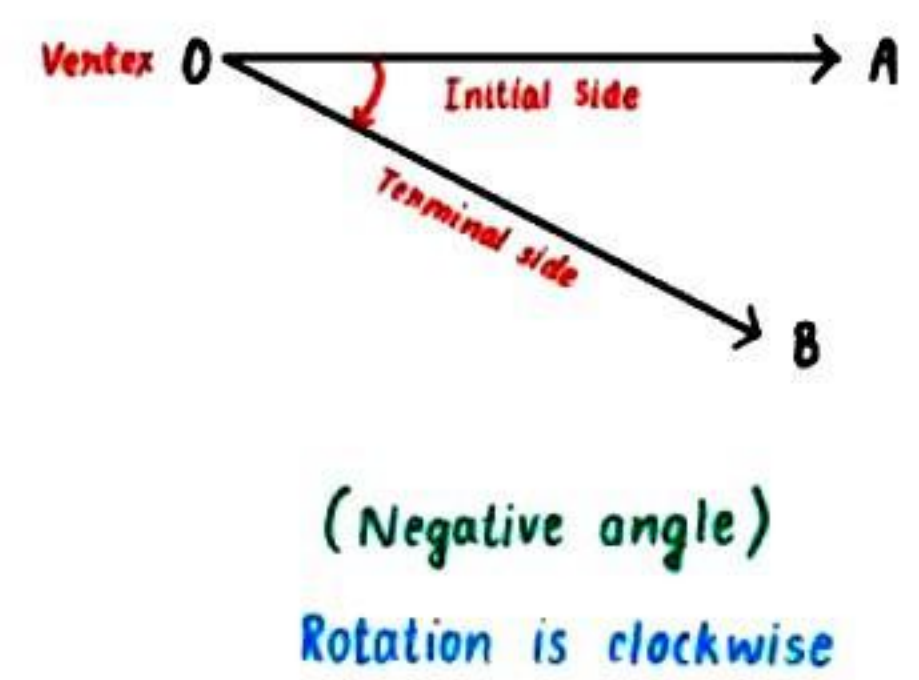
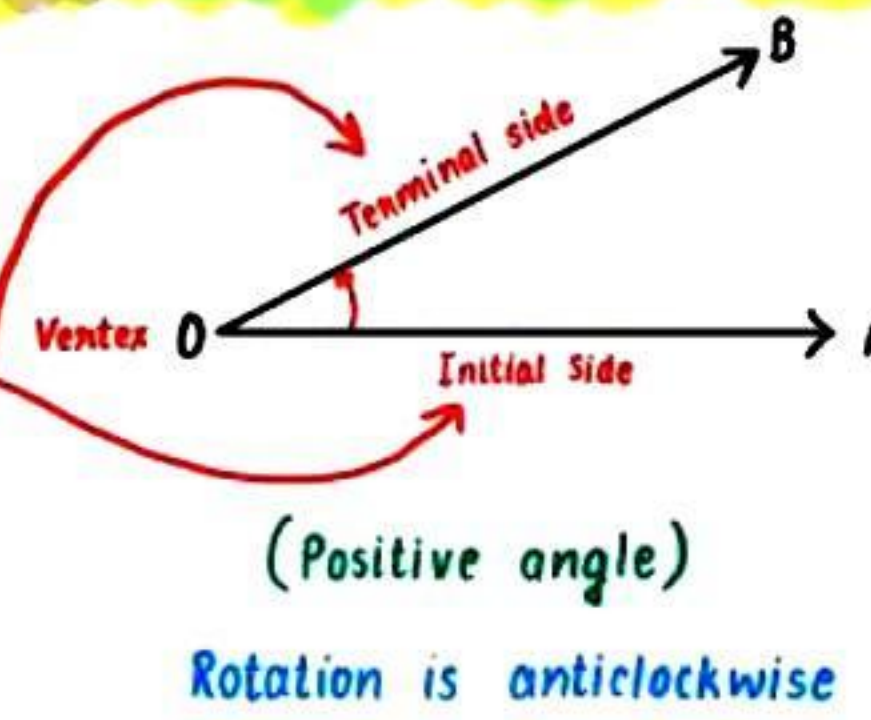


# TRIGONOMETRY FUNCTIONS

- ✓ Angle is a measure of rotation of a given ray about its initial point.
- ✓ The original ray is called the initial side. The final position of the ray after rotation is called terminal side.



♥  $1^\circ = 60'$     ♥  $1' = 60''$      $1^\circ$  (one degree) /  $1'$  (one minute) /  $1''$  (one second)

✓  $\theta = \frac{l}{r}$

$\theta$  = angle  
 $l$  = an arc of length  
 $r$  = radius of a circle

♥  $2\pi$  radian =  $360^\circ$   
 ♥  $\pi$  radian =  $180^\circ$

♥ Value of  $\pi = \frac{22}{7} = 3.14$

♥  $1$  radian =  $\frac{180^\circ}{\pi}$     ♥  $1^\circ = \frac{\pi}{180}$  radian

Radian measure =  $\frac{\pi}{180} \times$  Degree measure

Degree measure =  $\frac{180}{\pi} \times$  Radian measure

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$   
 $\sin(x-y) = \sin x \cos y - \cos x \sin y$   
 $\cos(x+y) = \cos x \cos y - \sin x \sin y$   
 $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$\cos(\pi - x) = -\cos x$

$\sin(\pi - x) = \sin x$

$\cos(\pi + x) = -\cos x$

$\sin(\pi + x) = -\sin x$

$\cos(2\pi - x) = \cos x$

$\sin(2\pi - x) = -\sin x$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$

$\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

♥  $\sin x = 0$  gives  $x = n\pi$ , where  $n \in \mathbb{Z}$

♥  $\cos x = 0$  gives  $x = (2n+1)\frac{\pi}{2}$ , where  $n \in \mathbb{Z}$

♥  $\sin x = \sin y$  implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$

♥  $\cos x = \cos y$  implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

♥  $\tan x = \tan y$  implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$   
 $\cos 2x = \cos^2 x - \sin^2 x$  or  $\frac{1 - \tan^2 x}{1 + \tan^2 x}$   
 $= 2 \cos^2 x - 1$   
 $= 1 - 2 \sin^2 x$   
 $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

$\sin 3x = 3 \sin x - 4 \sin^3 x$   
 $\cos 3x = 4 \cos^3 x - 3 \cos x$   
 $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$   
 $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$   
 $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$   
 $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

$2 \cos x \cos y = \cos(x+y) + \cos(x-y)$   
 $-2 \sin x \sin y = \cos(x+y) - \cos(x-y)$   
 $2 \sin x \cos y = \sin(x+y) + \sin(x-y)$   
 $2 \cos x \sin y = \sin(x+y) - \sin(x-y)$

✓ **Quadrantal angles** : All angles which are integral multiples of  $\frac{\pi}{2}$  are called quadrantal angles.

$\sin x = 0$  implies  $x = n\pi$ , where  $n$  is any integer.

$\cos x = 0$  implies  $x = (2n+1)\frac{\pi}{2}$ , where  $n$  is any integer.

$\operatorname{cosec} x = \frac{1}{\sin x}$ ,  $x \neq n\pi$ , where  $n$  is any integer.

$\sec x = \frac{1}{\cos x}$ ,  $x \neq (2n+1)\frac{\pi}{2}$ , where  $n$  is any integer.

$\tan x = \frac{\sin x}{\cos x}$ ,  $x \neq (2n+1)\frac{\pi}{2}$ , where  $n$  is any integer.

$\cot x = \frac{\cos x}{\sin x}$ ,  $x \neq n\pi$ , where  $n$  is any integer.

✓ **Trigonometric Equations** : Equations involving trigonometric functions of a variable are called trigonometric Equations.

✓ **Principal solutions** : The solutions of a trigonometric equation for which  $0 \leq x \leq 2\pi$  are called principal solutions.

✓ **General solution** : The expression involving integer 'n' which give all solutions of a trigonometric equation is called the general function.

✓ **Trigonometry Table** :

$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined	0
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not defined	-1	Not defined
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	-1	Not defined	1
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not defined	0	Not defined

✓ **Trigonometric functions in different quadrants** :

